Estimating Demand with Distance Functions: Parameterization in the Primal and Dual

Rolf Fare
Oregon State University, Corvallis, Oregon, USA
Shawna Grosskopf
Oregon State University, Corvallis, Oregon, USA
Kathy J. Hayes
Southern Methodist University, Dallas, Texas, USA
Dimitris Margaritis
Auckland University of Technology, Auckland, New Zealand

Abstract

Our purpose is to investigate the ability of different parametric forms to 'correctly' estimate consumer demands based on distance functions using Monte Carlo methods. Our approach combines economic theory, econometrics and quadratic approximation. We begin by deriving parameterizations for transformed quadratic functions which are linear in parameters and characterized by either homogeneity or which satisfy the translation property. Homogeneity is typical of Shephard distance functions and expenditure functions, whereas translation is characteristic of benefit/shortage or directional distance functions. The functional forms which satisfy these conditions and include both first and second order terms are the translog and quadratic forms, respectively. We then derive a primal characterization which is homogeneous and parameterized as translog and a dual model which satisfies the translation property and is specified as quadratic. We assess performance by focusing on empirical violations of the regularity conditions. Our analysis corroborates results from earlier Monte Carlo studies on the production side suggesting that the quadratic form more closely approximates the 'true' technology or in our context consumer preferences than the translog.

Keywords: Distance functions, demand, approximation, quadratic, translog, Monte Carlo.

1. Introduction

In this paper we analyze the consumer and consumer demand functions based on the duality between the expenditure and benefit functions. These functions are parameterized within the family of transformed (generalized) quadratic functions which are linear in the parameters and have a second order Taylor series interpretation. The combination of the quadratic structure and homogeneity or translation gives rise to parametric functional forms that include the translog and the quadratic. Thus in the primal (goods) space we parameterize the shortage function as quadratic and in the dual (price) space we parameterize the expenditure function as translog.

Much work has been done on defining appropriate functional forms for utility and demand functions.[1] These models include the translog model (Christensen, Jorgenson and Lau (1975)), the Rotterdam model (Theil (1975)), the almost ideal demand system (AIDS) (Deaton and Muellbauer (1980)), the quadratic almost ideal demand system (QUAIDS) (Banks, Blundell, and Lewbell (1997)) and the semi-flexible almost ideal demand system (Moschini (1998)). There have been similar analyses of inverse demand systems. These include the inverse translog (Christensen, Jorgenson, and Lau (1975)), the inverse Rotterdam model (Theil 1975, 1976; Barten and Bettendorf (1989)) the linear inverse demand system (Moschini and Vissa (1992); Eales and Unnevehr (1994)) and the inverse normalized quadratic (Holt and Bishop (2002)) .

Based on economic theory, a researcher cannot a priori choose one functional form over another. There are several approaches used in the literature to find the "best" functional form. Berndt, Darrough and Diewert (1977) compare functional forms based on goodness of fit tests. Of the three functional forms: the Generalized Leontief (GL), the Generalized Cobb-Douglas (GCD) and the Translog (TL), they concluded the translog was the best. Fisher, Fleissig and Serletis (2001) compare three locally flexible functional forms (GL, TL, and AIDS), three (effectively) globally regular functional forms (the Full Laurent model, QUAIDS, and generalized exponential form (G.E.F) and two globally flexible functional forms (the Fourier model and the Asymptotically Ideal Model (AIM)). Their comparison is based on statistical information criteria (AIC, SIC), elasticities

[1] Diewert (1971) defines flexible function forms to be those that can provide a local second order approximation to any twice continuously differentiable function.

of substitution and out-of-sample forecasts. Using a data set of quarterly US private consumption, prices and expenditures from 1960:1 to 1991:4, they find that global models perform better than locally flexible functional forms.

A second approach uses the fact that the properties of the demand functions derived from neoclassical preferences are known only in the region where they satisfy regularity conditions. The preferences of a rational consumer should satisfy monotonicity and quasiconvexity. At a particular combination of prices and income, locally flexible functional forms such as the translog (TL) and the Generalized Leontief (GL) can recover the elasticities with the appropriate choice of the model's parameters. However, they should satisfy regularity conditions at each possible value of income and prices. Knowing how large the regular region is can help support the choice of a functional form over another. Caves and Christensen (1980) compare the regular region of the GL and TL for two commodities and homothetic preferences. They conclude that the GL has larger regular region when the elasticity of substitution is small, and the opposite when it is large. Barnett and Lee (1985) using a Monte Carlo study showed that the regular region of locally flexible functional forms is relatively small.

A third approach uses a Monte Carlo study to focus on the accuracy of the demand model when the true elasticities of substitutions are known. Barnett and Choi (1989) find that (TL), GL and the Rotterdam model perform well in approximating the correct elasticities when the elasticities are similar and high. Barnett and Usui (2006) report that monotonicity violations are more likely to occur for the Normalized Quadratic model when elasticities of substitution are greater than unity. They also found that imposing curvature locally produces poor estimates of elasticities and smaller regular regions.

The dilemma researchers face is about the extent of structure one needs to impose on the empirical specification of the model without compromising the ability of data to reveal themselves information about consumer preferences. For example, imposing global curvature may induce spurious improvement in model-fit through monotonicity violations (see Barnett and Usui(2006)). The purpose of this study is to use duality theory to assess alternative parameterizations of functional forms for consumer demand systems. This paper is unique in offering primal and dual functional forms which satisfy translation and homogeneity properties. Rather than imposing globally or locally regularity conditions, such as inequality (monotonicity and curvature) constraints on the functional forms we let the data reveal consumer preferences. As the main problem with many functional forms is that they frequently reject neoclassical theory in empirical applications, we assess functional form performance by simply focusing on regularity violations. We estimate systems of demand equations derived from a translog specification of the expenditure function in the primal and a quadratic specification of the benefit function in the dual using survey data. To assess the robustness of our findings we carry out Monte Carlo simulations assuming that consumer preferences are generated from a CES indirect utility function.

2. Functional Forms

In this section we introduce a method for creating and choosing parametric functions for estimation in economics. The method takes into consideration several elements including: i) economic theory, ii) econometrics and iii) quadratic approximation.

Let $F : R^2 \to R$ be a smooth function from the plane to the line.[2] The second order Taylor's series approximation of $F(q)$ at $q^* \in R^2$ is given by

$$F(q) = F(q^*) + DF(q^*)(q - q^*) + \frac{(q - q^*)D^2 F(q^*)(q - q^*)}{2} \qquad (1)$$

where $DF(q^*)$ is the Jacobian matrix of F at q^*. In economics this function is frequently restricted to a transformed quadratic function (see Diewert (2002)) or a second order Taylor series approximation interpretation function (Färe and Sung (1986)) or a generalized quadratic (Chambers (1988)), i.e.,

$$F(q_1, q_2) = \varsigma^{-1}(a_0 + \sum_{i=1}^{2} a_i h(q_i) + \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij} h(q_i) h(q_j)) \qquad (2)$$

where $h : R \to R$ is twice differentiable and $\varsigma : R \to R$ has an inverse and a_i, a_{ij} are real constants. If $a_i \neq 0$ (i=1,2), $a_{ij} \neq 0$ (i,j=1,2) and $\varsigma^{-1} = q^{-1}$ then (2) reduces to a quasilinear function

[2] The arguments carry over to functions defined on R^N, N finite.

(Aczél (1966)), and if a_i = 0 (i=1,2), $a_i \neq 0$ (i,j=1,2) then (2) is a generalized quasi-quadratic function (Färe and Sung (1986)).

Writing (2) as

$$\varsigma \; F(q_1, q_2) = a_0 + \sum_{i=1}^{2} a_i h(q_i) + \sum_{i=1}^{2} \sum_{j=1}^{2} a_{ij} h(q_i) h(q_j) \tag{3}$$

shows that it is linear in the parameters a_i and a_{ij}, which is convenient for estimation. This form is familiar from work done by Diewert on exact and superlative index numbers.[3] Diewert (2002) uses the function in (3) "… to establish all of the superlative index number formulae that were derived in Diewert (1976)."

From economics we also know that by definition some aggregator functions are homogeneous in their variables (or subsets of their variables) and other aggregator functions satisfy the translation property by definition. Examples of homogeneous functions include Shephard's (1953) input distance function (linear homogeneous in inputs) and its dual cost function (linear homogeneous in input prices). Examples of aggregator functions that satisfy the translation property include the benefit function, shortage function (or directional distance function), see Luenberger (1995) and Chambers, Chung and Färe (1998).

Homogeneity (of degree one) is defined as

$$F(\lambda q) = \lambda F(q), \lambda > 0 \tag{4}$$

and the translation property is

$$F(q + ag) = F(q) + a, g \neq 0, a \in R \tag{5}$$

[3] In this literature an index is superlative if it is exact for a flexible aggregator function. An aggregator function is flexible if it provides a second-order approximation to a twice differentiable linearly homogeneous aggregator function. An index is exact for an aggregator function if economic optimizing behavior implies that the index can be defined as ratios of the aggregator function.

The last property tells us that if q is translated into $q+ag$, the value of the function at q changes to $F(q)+a$. The homogeneity property in (4) is a multiplicative or scaling condition, whereas the translation property in (5) is additive.

So the question at this point is what functional forms can we use for our linear in parameters aggregator function in (3) which also satisfies either homogeneity or translation? Färe and Sung (1986) take up the question for the homogeneous case. They show that there are two solutions to the functional equation generated by (3) and (4), namely, the translog

$$F(q_1, q_2) = a_0 + \sum_{i=1}^{2} a_i \ln q_i + \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij} \ln q_i \ln q_j \qquad (6)$$

and the quadratic mean of order ρ function

$$F(q_1, q_2) = (a_{11}q_1 + a_{22}q_2 + a_{12}q_1^{\rho/2}q_2^{\rho/2})^{1/2} \qquad (7)$$

with the appropriate restrictions on the parameters a_i, a_{ij}, ρ.

Turning to the case which combines (3) and translation, Färe and Lundberg (2006) prove that there are again only two functional forms which simultaneously satisfy these conditions, namely the quadratic function

$$F(q_1, q_2) = a_0 + \sum_{i=1}^{2} a_i q_i + \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij} q_i q_j \qquad (8)$$

and the function

$$F(q_1, q_2) = \frac{1}{2\lambda} \ln \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij} \exp(\lambda q_i)\exp(\lambda q_j) \qquad (9)$$

with appropriate restrictions on the parameters.

Chambers (1998) suggests these two functional forms for parameterizations of functions with the translation property; he takes λ=1/2 for the function in (9).[4]

A quick look at (6) and (8) reveals that these two functions have both first order parameters a_i and second order parameters a_{ij}, whereas (7) and (9) have only second order parameters a_{ij}. Since we are also interested in having a flexible functional form we focus on the specifications which have both first and second order terms, i.e., (6) and (8) - translog and quadratic.

Next we turn to the economic models that we will estimate and how the above discussion helps us to choose the appropriate specifications.

3. Relevant Consumer Theory

We now turn to the theoretical underpinnings of our model. Our aim is to derive a primal model with which we can estimate demand functions as functions of prices and a dual model with which the inverse demand functions are estimated as functions of quantities. To develop this model requires a duality theory between the benefit and expenditure functions together with a separability condition based on translation homotheticity.

With this theoretical structure we can apply the ideas of Section 2 and parameterize the estimating functions as translog for the primal model and as a quadratic function for the dual model.

Let $x = (x_1, x_2) \in R_+^2$ be a consumption bundle, where R_+^2 is the nonnegative orthant in the Cartesian product RxR.[5] Consumer preferences are summarized by the utility function

$$U : R_+^2 \to R. \tag{10}$$

We assume at the outset that $U(.)$ is quasi-concave, monotonic and continuous in $x \in R_+^2$.

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[4] Luenberger (1995) employs a version of the quadratic function in (8) in his exercise 14, p. 127.

[5] This may be extended to R_+^N, N finite.

Following Luenberger (1995)[6] the benefit (or directional input distance function) is defined in terms of the utility function as

$$b(x,u;g) = \max\{\beta : U(x_1 - bg_1, x_2 - bg_2) \geq u\} \qquad (11)$$

where $g = (g_1, g_2) \in R^2_+, g \neq 0$ is the directional vector.[7] The benefit function has two sets of properties: those inherited from the utility function and those derived from its definition.

Included among the inherited properties are concavity of $b(x,u;g)$ in x, $b(x,u;g)$ non-decreasing in x and non-increasing in u. Continuity of $b(x,u;g)$ in $x \in \textit{Interior } R^2_+$ follows from concavity.

From its definition it follows that $b(x,u;g)$ satisfies the translation property discussed in Section 2, in this case

$$b(x + ag, u; g) = b(x,u;g) + a \text{ if } (x + ag) \in R^2_+, a \in R \qquad (12)$$

and it is homogeneous of degree -1 in g, i.e.,

$$b(x,u;\lambda g) = \lambda^{-1} b(x,u;g), \lambda > 0 . \qquad (13)$$

Deaton (1979)[8] introduced Shephard's (1953) input distance function to consumer theory and defined it in terms of the utility function as

$$d(x,u) = \max\{\lambda : U(x/\lambda) \geq u\}. \qquad (14)$$

Following Chambers, Chung and Färe (1996) one can show that the benefit or shortage function is related to the Shephard distance functions as

$$b(x,u;g) = 1 - 1/d(x,u), \qquad (15)$$

[6] See also Luenberger (1992) and Chambers, Chung and Färe (1996).
[7] In the efficiency analysis literature, g is the direction in which technical efficiency is evaluated (see Chambers, Chung and Färe (1996)).
[8] See also Deaton and Muellbauer (1980).

where g is taken to be equal to x, i.e., g=x.[9] Having established the relationship between Deaton's (1979) approach using Shephard distance functions and our approach using the benefit or directional distance function, hereafter we follow Luenberger (1995) and take the direction vector g equal to (1,1). This yields solution values which may be interpreted as the number of units of x_1 and x_2 which can be 'saved' and still achieve utility level u, rather than as the proportional factor by which they could be reduced when g=x. For notational simplicity we also suppress the direction vector, i.e., we will write $b(x,u)$ rather than $b(x,u;1,1)$.

The next step is to create a duality condition between the expenditure function and the benefit function. Let $p=(p_1,p_2)\in R_+^2$ denote the unit prices of consumption goods x_1 and x_2. Since we have assumed the utility to be monotonic, it suffices for these prices to be nonnegative.

The expenditure function is defined in terms of the benefit function as[10]

$$e(p,u)=\min_{x\in R_+^2}\{px-b(x,u)pI_2\} \tag{16}$$

where $I_2=(1,1)$.

Since under very mild conditions (see Luenberger (1995))

$$U(x)\geq u \Leftrightarrow b(x,u)\geq 0, \tag{17}$$

expression (16) may also be written in terms of the utility function

$$e(p,u)=\min_{x\in R_+^2}\{px:U(x)\geq u\}.^{11} \tag{18}$$

Since in general

$$e(p,u)\leq px-b(x,u)pI_2, \tag{19}$$

[9] This is consistent with the Farrell measure of input technical efficiency in the production efficiency literature.
[10] See Luenberger (1995) and Chambers, Chung and Färe (1996).
[11] The properties of the expenditure function are well-known; important for our purposes is the fact that the expenditure function is homogeneous of degree +1 in prices.

the dual[12] to (16) is

$$b(x,u) = \min_{p \in R_+^2} \{ \frac{px - e(p,u)}{pI_2} \}. \tag{20}$$

The demand and inverse demand functions now follow by applying Shephard's primal and dual lemmata. Specifically

$$\nabla_p e(p,u) = x(p,u) \tag{21}$$

$$\nabla_x b(x,u) = p(x,u) / pI_2. \tag{22}$$

Both of these expressions are functions of utility levels, thus their parametric forms can not be directly estimated. In order to remove the dependence on utility we could introduce the assumption that preferences are translation homothetic.[13] When preferences are translation homothetic, the expenditure and benefit function can be written as

$$e(p,u) = \hat{e}(p,1) - H(u) \tag{23}$$

and

$$b(x,u) = \hat{b}(x) + H(u). \tag{24}$$

Thus under this condition, expressions (19) and (20) are independent of utility and take the simplified forms

$$\nabla_p e(p,u) = x(p) \tag{25}$$

$$\nabla_x b(x,u) = p(x). \tag{26}$$

Rather than assuming that preferences are translation homothetic, here we follow Lewbel and Pendakur (2006) and proxy utility with income and estimate what they call pseudo-Marshallian demands.

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[12] For the duality statements and their proofs see Luenberger (1992, 1995) and Chambers, Chung and Färe (1996). Note that we have imposed convexity and closedness on the ‘at least as good as set’, so duality applies.

[13] See Chambers and Färe (1998) for details concerning this concept. Färe, Martins-Filho and Vardanyan (2006) applied a similar procedure to production economics.

Another approach to this problem is taken by Wong and McLaren (2005) who remove the utility from a Shephard distance function $d(x,u)$ by creating the inverse $u = d^{-1}(x,1)$ and substituting this into the original distance function. In the next section we parameterize the expenditure and benefit functions using the results from Section 2.

4. Parameterizing the expenditure and benefit functions

Using our notation from the previous section, we begin by parameterizing the expenditure function as

$$\ln e(p,y) = a_0 + \sum_{i=1}^{I} a_i \ln p_i + a_y \ln y + \frac{1}{2}\sum_{i=1}^{I}\sum_{j=1}^{I} a_{ij} \ln p_i \ln p_j + \sum_{i=1}^{I} a_{yi} \ln p_i \ln y + a_{yy} (\ln y)^2 \quad (27)$$

where $a_{ij} = a_{ji}$.

Applying Shephard's lemma yields the compensated demands:

$$\frac{\partial e(p,y)/\partial p_i}{e(p,y)} = a_i / p_i + \frac{1}{2}\sum_{j=1}^{I} a_{ij} \ln p_j / p_i + a_{yi} \ln y / p_i, i = 1,\ldots,I \quad (28)$$

$$= x_i / e(p,y) \quad (29)$$

Multiplying both sides by p_i yields share equations

$$\frac{p_i x_i}{e(p,y)} = s_i = a_i + \frac{1}{2}\sum_{j=1}^{I} a_{ij} \ln p_j + a_{yi} \ln y \quad (30)$$

Imposing linear homogeneity requires the following restrictions on the parameters:

$$\sum_{i=1}^{L} a_i = 1, \text{ and } \sum_{i=1}^{L} a_{yi} = 0 = \sum_{j=1}^{L} a_{ij}\,. \quad (31)$$

Turning to the quadratic specification of the benefit function, we have

$$b(x,y) = a_0 + \sum_{i=1}^{I} a_i x_i + a_y \ln y + \frac{1}{2}\sum_{i=1}^{I}\sum_{j=1}^{I} a_{ij} x_i x_j + \sum_{i=1}^{I} a_{yi} x_i \ln y + a_{yy} (\ln y)^2 \tag{32}$$

Differentiating with respect to good *i* yields

$$\frac{\partial b(x,y)}{\partial x_i} = \frac{p_i}{\sum_{i=1}^{L} p_i} = a_i + \frac{1}{2}\sum_{j=1}^{I} a_{ij} x_j + a_{yi} y, i = 1,..., I \tag{33}$$

Imposing translation requires the following restrictions on the parameters:

$$\sum_{i=1}^{L} a_i = 1, \text{ and } \sum_{i=1}^{L} a_{yi} = 0 = \sum_{j=1}^{L} a_{ij} \ . \tag{34}$$

5. An Empirical Illustration

We use data from the 1987-1988 Nationwide Food Consumption Survey (NFCS) to estimate the demand systems derived from the translog and quadratic specifications of the expenditure function and the benefit function, respectively. The data contain information recorded over a seven day period from 4243 households that participated in the USDA survey during April 1987 to May 1988. The sample is restricted to households that reported positive income and had at least one member consuming 10 or more meals from the household food supply (see Huang and Lin (2000) for more details). As in Huang and Lin (2000) we focus on foods consumed at home aggregated into 13 composite categories: beef, pork, poultry, other meat, fish, dairy products, cereal, bread, fats and oil, eggs, vegetables, fruits, and juice. We assume that home food consumption is separable from the demand for other goods in each household's budget.

In table 1 we present the summary statistics for the data. The average annual income for the households in the data set is under $28,000. On average these households spent just over $3,200 on food consumption at home.[14] Fish and bread were the most expensive food categories per unit of

[14] Food consumption at home represents on average about 75 percent of the total household food budget (see Huang and Lin (2000)). We focus on food consumption at home as detailed information on food consumed away from home is not provided in the NFCS data.

food. Half of the at home food budget was spent on dairy products, vegetables and beef. While almost all households consumed these three types of foods, about half of them did not consume any fish at home.

Tables 2 and 3 present SUR estimates of the systems of budget shares and normalized (inverse) demand functions associated with the expenditure and benefit functions, respectively, for the 13 food categories. To avoid singularity problems we have dropped one equation during estimation while imposing the linear homogeneity or translation restrictions, as appropriate. The omitted food category is fish – as we noted above about half of the households did not report any fish consumption during the survey period. Invoking the separability assumption we used total food expenditure at home as the income variable in (30) and (33). To control for demographic characteristics on household consumption patterns, we added household size and the education level of the female head of household in the estimated equations.

The results of the translog model are shown in Table 2. All own price marginal effects on budget shares are positive as required by the curvature property of the expenditure function. The largest own effects are estimated for bread and vegetables. Evidence of strong complementary effects is profound. Not surprisingly, the dairy category is a complement to all other categories. Income effects are significantly positive for dairy products, fruit and vegetables and negative for the other food categories except pork for which the effect is positive but not statistically significant. We find that larger household size leads to a larger budget share spent on meats but not fish. Also less is spent on vegetables, fruits and juices as household size gets bigger. For those households with more educated female head, the expenditures shares are larger for those goods.

Table 3 shows the results for the benefit function. We estimate (inverse) demand equations where the dependent variable is normalized prices. All own quantity marginal effects are negative as required by the curvature property of the benefit function. The coefficients are numerically very small indicating large price elasticities. The smallest (in absolute value) own quantity effects are estimated for vegetables, fruit and juice. Again evidence of strong complementary effects is profound albeit not to the extent found in the case of the translog.[15] Dairy products appear to be a substitute for many goods but not for fruits and vegetables. Higher incomes lead to higher prices paid for dairy products, vegetables and fruit and lower prices for pork, other meats, cereals, fats, eggs and juice. Household size influences the prices paid for fish and dairy negatively and has a positive impact on the marginal willingness to pay for cereals, eggs, fruits and juice. More highly educated female heads are willing to pay more at the margin for poultry but less for dairy products, cereals, eggs and juice.

A comparison of the empirical results for the translog and quadratic functions gives no prima facie evidence for choosing one form over the other. Both specifications appear to fit the data quite well and satisfy the curvature conditions adhered to by consumer theory. However, as reported in table 4, the number of monotonicity violations is much larger for the translog function than the quadratic function. In fact, it appears that the translog encounters a substantial number of monotonicity violations for this data set thereby bringing to question the validity of inferences on consumer behavior.[16] We investigate the ability of the two functional forms to satisfy the regularity conditions of consumer behavior further by conducting a Monte Carlo study.

We follow an experimental design similar to that used in the Monte Carlo study by Barnett and Usui (2006). The following procedure is adopted: (1) a vector of prices is created based on observed data plus an error term generated from a multivatiate normal with mean zero and covariance matrix equal to a multiple, $\mu \in [0,1]$, of the actual price covariance matrix while ensuring that prices remain nonnegative; (2) we feed the price data along with total expenditure (y) to demand equations derived from a CES indirect utility function to generate data on quantities demanded; (3) we use the price, quantity and income data to estimate the budget share and normalized price equations derived from the translog and quadratic function specifications of the expenditure function and benefit function, respectively.

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[15] Note that valuations of complementary relationships across different types of foods may be of interest for welfare analysis purposes – e.g. enhance the effectiveness of government food programs.
[16] Basmann, Molina and Slottje (1983) have drawn attention to the inference problem that arises in the estimation of consumer demand systems when regularity conditions are violated. See also Barnett and Usui (2006).

We assess the two functional forms by estimating systems of demand equations for six composite food categories: meats, eggs and dairy, cereals and bread, fruits and vegetables, juice, fats and oil.[17] The CES indirect utility function with six goods is given by:

$$V(p,y) = y\left[\sum_{i=1}^{6} p_i^r\right]^{-1/r} \quad \text{where } r = \rho/(\rho-1)\,, \quad \rho \le 1$$

Using Roy's identity to the CES indirect utility function, we can derive Marshallian demand functions (see Barnett and Usui (2006)) as:

$$q_i(p,y) = yp_i^{r-1} / \sum_{j=1}^{6} p_j^r$$

Quantities are generated based on the generated prices for given income and values of ρ. We choose values of ρ that give rise to a sufficiently wide range of elasticities of substitution, defined as $\sigma = 1/(1-\rho)$. Using the simulated price and quantity data, we calculate the expenditures shares and normalized prices required to estimate the systems of equations given in (30) and (33) above. The number of Monte Carlo replications is 1000 so we estimate the two equations 1000 times and check for regularity violations in each case. We repeat the same experiment using values of σ ranging from 0.2 to 8. In table 8, we report regularity violations for elasticitity of substitution values set to 0.2, 2 and 5. While both functional forms satisfy the curvature condition when the data is generated with elasticities of substitution less than one, the translog fails to produce meaningful results for values of σ>1. In all cases we find that the quadratic encounters far less monotonicity violations than the translog.[18]

[17] SUR estimates of the budget shares and normalized prices equations using observed data for the six composite goods categories are reported in Tables 5 and 6, respectively. Both functions satisfy the curvature conditions. Monotonicity violations are reported in Table 7.

[18] While not reported here, we experimented further using the Diewert reciprocal indirect utility function (with homothetic preferences) and the translog reciprocal indirect utility function to generate quantity data. Again, the evidence indicates that the quadratic specification performs better than the translog model.

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Table 1: Summary Statistics: 1987-1988 US Food Consumption Survey (N=4243)

	beef	pork	poultry	other meats	fish	dairy	cereals	bread	fats	eggs	veggies	fruits	juice
Price	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13
Mean	1.93	2.14	1.41	2.18	2.88	0.84	1.82	2.71	1.11	0.56	0.74	0.80	0.44
Median	1.73	1.93	1.09	1.96	2.43	0.57	1.62	2.31	1.01	0.53	0.65	0.71	0.39
Std. Dev.	0.90	0.99	1.04	0.99	1.67	0.90	1.06	1.93	0.49	0.24	0.47	0.48	0.24
Skewness	2.52	2.68	2.20	1.76	3.23	4.54	1.56	4.59	2.14	6.57	8.78	3.46	2.86
Kurtosis	14.15	16.90	11.17	8.11	36.19	36.11	7.49	37.62	17.72	77.86	159.05	26.46	17.24
Quantity	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13
Mean	3.56	2.02	2.89	1.17	1.02	20.21	2.53	2.37	1.61	1.23	12.13	6.25	4.94
Median	2.75	1.00	2.00	0.75	0.25	15.18	1.67	1.90	1.18	0.88	10.08	4.14	3.29
Std. Dev.	3.64	3.19	3.78	1.63	2.24	17.63	2.99	1.97	1.60	1.24	9.43	8.20	6.14
Skewness	2.07	9.28	3.62	3.24	7.19	1.88	3.51	2.61	2.40	2.26	1.82	4.31	3.47
Kurtosis	10.40	237.91	29.32	20.74	89.88	7.72	24.55	17.56	13.56	11.89	9.40	35.63	31.39
Share	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10	s11	s12	s13
Mean	0.11	0.06	0.06	0.04	0.04	0.23	0.06	0.10	0.03	0.01	0.14	0.08	0.04
Median	0.09	0.04	0.04	0.02	0.01	0.19	0.05	0.09	0.02	0.01	0.13	0.06	0.03
Std. Dev.	0.10	0.07	0.06	0.05	0.07	0.16	0.06	0.08	0.02	0.01	0.09	0.08	0.04
Skewness	1.23	1.82	2.10	2.69	2.94	1.19	2.84	2.14	2.10	4.68	1.45	2.29	2.71
Kurtosis	4.82	8.05	10.18	15.12	14.89	4.47	24.92	11.74	11.97	54.42	7.30	11.83	17.16

	FOODEXP	INCOME	FHDED	MHDED	HHSIZE
Mean	61.72	27481	11.25	9.42	2.79
Median	51.81	22501	12	12	2
Maximum	557.30	330300	18	18	12
Minimum	1.39	50	0	0	1
Std. Dev.	44.18	23468	4.43	6.16	1.45
Skewness	2.29	3.26	-1.28	-0.54	0.91
Kurtosis	13.13	26.91	4.37	1.88	4.07

Notes:

PRICE = dollars per pound
SHARE= budget share
FOODEXP = household food expenditure at home over a 7-day period
INCOME = household annual income
FHDED = female head of household education
MHDED = female head of household education
HHSIZE = household size measured in 21-meal equivalence.

Table 2: Expenditure Function – SUR Estimates

Dependent Variable: Expenditure Share

Coef	Est.	z-stat	Coef	Est.	z-stat	Coef	Est.	z-stat	Coef	Est.	z-stat
a11	0.129	18.49	a44	0.038	10.32	a810	-0.005	-6.29	ahs5	-0.009	-9.18
a12	0.002	0.35	a45	-.0003	-0.14	a811	-0.035	-10.92	ahs6	0.007	4.46
a13	-0.001	-0.27	a46	-0.014	-6.08	a812	-0.012	-4.38	ahs7	0.008	10.54
a14	-0.007	-2.00	a47	0.002	0.89	a813	-0.010	-4.57	ahs8	0.007	8.36
a15	0.007	1.78	a48	-0.006	-2.47	a99	0.037	20.88	ahs9	0.0002	0.70
a16	-0.054	-13.95	a49	-0.002	-1.33	a910	-0.003	-2.95	ahs10	0.001	4.04
a17	-0.008	-2.60	a410	-0.002	-1.79	a911	-0.006	-3.53	ahs11	-0.007	-6.81
a18	-0.027	-7.50	a411	-0.007	-2.26	a912	-0.001	-1.03	ahs12	-0.015	-14.23
a19	-0.003	-1.87	a412	-0.002	-0.58	a913	0.006	4.30	ahs13	-0.001	-2.32
a110	-0.002	-1.39	a413	0.003	1.40	a1010	0.026	19.57	aef1	-0.001	-3.78
a111	-0.023	-5.17	a55	0.050	11.99	a1011	-0.001	-0.87	aef2	-0.001	-4.48
a112	-0.013	-3.21	a56	-0.025	-8.36	a1012	-0.001	-1.37	aef3	0.0002	0.69
a113	-.0004	-0.13	a57	-0.008	-3.36	a1013	-0.001	-1.25	aef4	-0.001	-4.88
a22	0.020	3.78	a58	-0.005	-1.68	a1111	0.178	31.55	aef5	-4E-05	-0.17
a23	0.001	0.52	a59	-0.004	-2.83	a1112	-0.022	-6.20	aef6	0.0003	0.60
a24	-0.002	-0.62	a510	-0.001	-0.98	a1113	-0.004	-1.46	aef7	-0.0003	-1.65
a25	0.003	0.86	a511	0.003	0.88	a1212	0.081	17.96	aef8	-0.0003	-1.55
a26	-0.016	-5.31	a512	-0.004	-1.22	a1213	-0.004	-1.80	aef9	0.0002	2.16
a27	-0.002	-0.58	a513	-0.007	-3.07	a1313	0.025	9.17	aef10	-0.0001	-2.91
a28	-0.007	-2.27	a66	0.279	50.93	ay1	0.006	2.19	aef11	0.001	3.83
a29	-.0001	-0.08	a67	-0.013	-5.61	ay2	0.001	0.62	aef12	0.002	7.98
a210	-.0004	-0.36	a68	-0.039	-14.49	ay3	-0.005	-3.00	aef13	0.0002	1.54
a211	-0.011	-2.78	a69	-0.008	-7.20	ay4	-0.004	-2.96	a1	0.042	4.07
a212	0.006	1.68	a610	-0.005	-6.98	ay5	0.020	9.40	a2	0.054	6.59
a213	0.007	2.46	a611	-0.054	-16.20	ay6	0.012	3.48	a3	0.073	10.62
a33	0.039	12.88	a612	-0.027	-8.45	ay7	-0.013	-8.00	a4	0.046	7.66
a34	-0.002	-0.73	a613	-0.012	-6.43	ay8	-0.025	-13.74	a5	-0.038	-4.76
a35	-0.009	-3.45	a77	0.055	19.83	ay9	-0.003	-3.88	a6	0.256	20.03
a36	-0.012	-4.61	a78	-.0002	-0.09	ay10	-0.004	-8.53	a7	0.086	13.69
a37	-0.007	-3.27	a79	-0.007	-5.84	ay11	0.007	3.00	a8	0.112	15.82
a38	0.003	1.17	a710	-0.002	-3.50	ay12	0.016	6.95	a9	0.043	15.00
a39	-.0004	-0.39	a711	-0.012	-4.46	ay13	-0.007	-5.65	a10	0.037	20.53
a310	-0.002	-3.28	a712	0.003	1.08	ahs1	0.004	3.31	a11	0.170	19.38
a311	-0.005	-1.72	a713	.0001	0.03	ahs2	0.003	2.81	a12	0.044	5.12
a312	-0.003	-1.22	a88	0.152	41.50	ahs3	-0.001	-0.79	a13	0.076	15.03
a313	-0.002	-1.27	a89	-0.008	-5.84	ahs4	0.003	4.45			

Notes:
1=Beef; 2=Pork; 3=Poultry; 4=Other Meats; 5=Fish; 6=Dairy; 7=Cereals; 8=Bread; 9=Fats and oil; 10=Eggs; 11=Vegetables; 12=Fruits; 13=Juice; y=food expenditure; hs=household size; ef=female-head education level.

Table 3: Benefit Function – SUR Estimates

Dependent Variable: Normalized Price

Coef	Est. X100	z-stat	Coef	Est. X100	z-stat	Coef	Est. X100	z-stat	Coef	Est. X100	z-stat
a11	-0.169	-4.69	a44	-0.580	-7.08	a810	-0.060	-2.33	ahs5	-0.589	-6.39
a12	0.032	1.12	a45	0.032	0.62	a811	-0.032	-1.39	ahs6	-0.421	-8.42
a13	0.014	0.52	a46	0.051	4.82	a812	-0.021	-0.92	ahs7	0.552	8.17
a14	0.003	0.07	a47	0.177	3.9	a813	-0.099	-4.97	ahs8	0.136	1.43
a15	0.045	1.18	a48	0.197	2.74	a99	-0.358	-6.69	ahs9	-0.048	-1.4
a16	-0.016	-1.79	a49	-0.003	-0.08	a910	0.005	0.2	ahs10	0.050	2.72
a17	0.026	0.81	a410	0.004	0.16	a911	0.036	3.5	ahs11	0.035	1.26
a18	-0.080	-1.66	a411	0.047	2.78	a912	0.026	2.73	ahs12	0.152	4.53
a19	0.083	3.73	a412	0.007	0.41	a913	0.018	1.59	ahs13	0.124	6.77
a110	0.024	1.93	a413	0.043	2.59	a1010	-0.151	-4.79	aef1	0.026	1.97
a111	-0.003	-0.23	a55	-0.382	-3.51	a1011	0.030	5.47	aef2	0.015	1.02
a112	0.019	1.52	a56	0.027	1.84	a1012	-0.001	-0.13	aef3	0.039	2.53
a113	0.022	2.1	a57	0.053	1.09	a1013	0.007	1.12	aef4	0.008	0.55
a22	-0.431	-9.85	a58	0.202	2.83	a1111	-0.090	-10.32	aef5	-0.003	-0.10
a23	0.091	3.19	a59	0.041	1.33	a1112	-0.007	-1.02	aef6	-0.031	-2.40
a24	-0.041	-1.03	a510	0.019	1.14	a1113	0.013	2.51	aef7	-0.039	-2.30
a25	-0.045	-1.10	a511	0.018	0.94	a1212	-0.075	-7.72	aef8	0.012	0.52
a26	0.021	2.12	a512	0.016	0.78	a1213	0.001	0.26	aef9	-0.004	-0.5
a27	0.082	2.32	a513	-2E-07	0.00	a1313	-0.078	-11.51	aef10	-0.017	-3.90
a28	0.187	3.49	a66	-0.138	-15.82	ay1	0.001	0.65	aef11	-0.006	-0.88
a29	0.043	1.77	a67	0.055	4.85	ay2	-0.013	-6.26	aef12	0.015	1.76
a210	0.020	1.46	a68	0.027	1.65	ay3	-0.002	-1.09	aef13	-0.016	-3.43
a211	0.013	0.96	a69	0.019	3.19	ay4	-0.019	-8.17	a1	9.815	55.66
a212	0.008	0.54	a610	0.006	2.03	ay5	-0.032	1.01	a2	11.109	57.08
a213	0.023	1.98	a611	-0.023	-4.95	ay6	0.055	32.84	a3	6.891	33.96
a33	-0.576	-15.2	a612	-0.020	-3.71	ay7	-0.017	-7.15	a4	11.489	56.68
a34	0.064	1.85	a613	0.000	-0.13	ay8	-0.001	-0.26	a5	15.735	52.00
a35	-0.025	-0.62	a77	-0.843	-14.8	ay9	-0.009	-7.11	a6	3.855	22.65
a36	-0.008	-0.82	a78	0.328	5.47	ay10	-0.006	-8.21	a7	8.932	39.77
a37	-0.052	-1.59	a79	0.057	2.01	ay11	0.011	10.43	a8	13.289	41.98
a38	0.322	6.76	a710	0.050	3.26	ay12	0.005	4.5	a9	5.894	53.28
a39	0.094	4.63	a711	0.010	0.66	ay13	-0.002	-3.5	a10	3.081	52.41
a310	0.047	4.25	a712	0.032	1.98	ahs1	0.014	0.27	a11	3.863	41.17
a311	-0.013	-0.99	a713	0.025	1.87	ahs2	0.045	0.77	a12	3.661	32.33
a312	0.016	1.17	a88	-0.911	-7.31	ahs3	0.070	1.16	a13	2.385	39.44
a313	0.026	2.54	a89	-0.060	-1.31	ahs4	-0.127	-2.06			

Notes:
1=Beef; 2=Pork; 3=Poultry; 4=Other Meats; 5=Fish; 6=Dairy; 7=Cereals; 8=Bread; 9=Fats and oil; 10=Eggs; 11=Vegetables; 12=Fruits; 13=Juice; y=food expenditure; hs=household size; ef=female-head education level.

Table 4: Monotonicity Violations
SUR Estimates

Variable	Translog					Quadratic Function				
	MVs	Mean Predict	Std. Dev.	Min Predict	Max Predict	MVs	Mean Predict	Std. Dev.	Min Predict	Max Predict
Beef	4	-0.0096	0.0076	-0.0182	-0.0029	0				
Pork	0					1	-0.0998	.	-0.0998	-0.0998
Poultry	0					7	-0.0293	0.0259	-0.0704	-0.0046
Other	3	-0.0006	0.0004	-0.0011	-0.0003	0				
Fish	59	-0.0147	0.0135	-0.0539	-0.0002	1	-0.0146	.	-0.0146	-0.0146
Dairy	9	-0.0256	0.0146	-0.0434	-0.0038	14	-0.0092	0.0104	-0.0397	-0.0005
Cereals	20	-0.0092	0.0080	-0.0250	0.0000	4	-0.0123	0.0039	-0.0163	-0.0080
Bread	27	-0.0149	0.0201	-0.0999	-0.0002	0				
Fats	6	-0.0050	0.0039	-0.0100	-0.0002	0				
Eggs	58	-0.0024	0.0023	-0.0103	0.0000	0				
Veggies	6	-0.0232	0.0173	-0.0432	-0.0045	0				
Fruits	28	-0.0152	0.0156	-0.0552	0.0000	0				
Juice	4	-0.0020	0.0016	-0.0035	-0.0001	1	-0.0153	.	-0.0153	-0.0153
Total	224					28				

Note:
MVs=number of monotonicity violations
Predict = predicted values if dependent variable estimate <0.

Table 5: Expenditure Function
SUR Estimates

Equation	Coef.	Z-stat	Equation	Coef.	z-stat
s1			s4		
log_p1	0.105521	16.78	log_p1	-0.0137	-4.25
log_p2	-0.00674	-1.65	log_p2	-0.00225	-0.76
log_p3	-0.0076	-2.45	log_p3	-0.00315	-1.48
log_p4	-0.0137	-4.25	log_p4	0.036408	10.52
log_p5	-0.00085	-0.17	log_p5	0.0022	0.75
log_p6	-0.07664	-20.62	log_p6	-0.01951	-8.86
log_y	0.009099	3.4	log_y	-0.0036	-2.41
hhsize	0.002435	1.94	hhsize	0.002917	4.2
fheadeduc	-0.00118	-3.59	fheadeduc	-0.00087	-4.81
_cons	0.038143	3.87	_cons	0.041926	7.5
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s2			s5		
log_p1	-0.00674	-1.65	log_p1	-0.00085	-0.17
log_p2	0.020664	4.14	log_p2	0.01496	3.78
log_p3	-0.00133	-0.49	log_p3	-0.00738	2.31
log_p4	-0.00225	-0.76	log_p4	0.0022	0.75
log_p5	0.01496	3.78	log_p5	0.09826	9.39
log_p6	-0.02531	-8.41	log_p6	-0.10719	-21.66
log_y	0.002782	1.33	log_y	-0.02111	-5.16
hhsize1	0.002622	2.69	hhsize	-0.01014	-5.25
fheadeduc	-0.00114	-4.48	fheadeduc	0.00264	5.20
_cons	0.038525	4.95	_cons	0.51489	34.29
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s3			s6		
log_p1	-0.0076	-2.45	log_p1	-0.07664	-20.62
log_p2	-0.00133	-0.49	log_p2	-0.02531	-8.41
log_p3	0.037104	12.56	log_p3	-0.01764	-7.05
log_p4	-0.00315	-1.48	log_p4	-0.01951	-8.86
log_p5	-0.00738	2.31	log_p5	-0.10719	-21.66
log_p6	-0.01764	-7.05	log_p6	0.246291	46.15
log_y	-0.00409	-2.27	log_y	0.01692	4.71
hhsize	-0.00129	-1.53	hhsize	0.003449	2.06
fheadeduc	0.000189	0.86	fheadeduc	0.000358	0.82
_cons	0.073387	11.03	_cons	0.293129	21.82
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Note:
Composite food categories are as follows: 1 = meats, 2 = eggs and dairy, 3 = cereals and bread, 4 = fruits and vegetables, 5 = juice, 6 = fats and oil.

Table 6: Benefit Function
SUR Estimates

Equation	Coef.	Z-stat	Equation	Coef.	Z-stat
p1/sum_p			p4/sum_p		
x_1	-0.0030	-5.38	x_1	0.0003	0.58
x_2	0.0010	2.22	x_2	0.0010	1.63
x_3	0.0011	2.71	x_3	0.0030	5.53
x_4	0.0003	0.58	x_4	-0.0072	-6.44
x_5	0.0008	1.47	x_5	0.0018	2.29
x_6	-0.0002	-1.5	x_6	0.0011	6.34
y	0.0000	-0.84	y	-0.0003	-9.86
hhsize	0.0028	3.31	hhsize	0.0019	1.97
fheadeduc	0.0002	0.87	fheadeduc	-0.0001	-0.35
_cons	0.1685	58.68	_cons	0.1967	60.04
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p2/sum_p			p5/sum_p		
x_1	0.0010	2.22	x_1	0.0008	1.47
x_2	-0.0060	-9.14	x_2	0.0005	0.84
x_3	0.0029	6.58	x_3	0.0013	2.20
x_4	0.0010	1.63	x_4	0.0018	2.29
x_5			x_5	-0.0051	-18.74
x_6	0.0006	4.03	x_6	0.0006	3.03
y	-0.0003	-9.04	y	-0.0001	2.78
hhsize	0.0040	4.39	hhsize	-0.0056	-4.27
fheadeduc	0.0001	0.23	fheadeduc	-0.0003	-0.82
_cons	0.1894	62.18	_cons	0.2653	59.33
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p3/sum_p			p6/sum_p		
x_1	0.0011	2.71	x_1	-0.0002	-1.5
x_2	0.0029	6.58	x_2	0.0006	4.03
x_3	-0.0084	-14.3	x_3	0.0001	0.83
x_4	0.0030	5.53	x_4	0.0011	6.34
x_5	0.0013	2.20	x_5	0.0006	3.03
x_6	0.0001	0.83	x_6	-0.0022	-16.2
y	-0.0001	-3.02	y	0.0008	31.72
hhsize	0.0026	2.69	hhsize	-0.0056	-6.96
fheadeduc	0.0006	2.46	fheadeduc	-0.0005	-2.33
_cons	0.1166	36	_cons	0.0635	23.32
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Note:
Composite food categories are as follows: 1 = meats, 2 = eggs and dairy, 3 = cereals and bread, 4 = fruits and vegetables, 5 = juice, 6 = fats and oil.

Table 7: Monotonicity Violations
SUR Estimates

	Translog					Quadratic Function				
	MVs	Mean Predict	Std. Dev.	Min Predict	Max Predict	MVs	Mean Predict	Std. Dev.	Min Predict	Max Predict
Group1	9	-0.012	0.007	-0.025	-0.002	0				
Group2	0					1	-0.128	.	-0.128	-0.128
Group3	0					5	-0.047	0.045	-0.108	0.000
Group4	4	-0.002	0.002	-0.005	0.000	0				
Group5	18	-0.231	0.209	-0.738	-0.047	0				
Group6	11	-0.016	0.018	-0.054	-0.002	9	-0.016	0.019	-0.063	0.000
Total	42					15				

Note:
MVs=number of monotonicity violations
Predict = predicted values if dependent variable estimate <0.

Table 8: Monotonicity Violations
SUR Estimates

Variable	r=0.8 (σ=0.2) Translog Violations	r=0.8 (σ=0.2) Quadratic Violations	r=-1 (σ=2) Translog Violations	r=-1 (σ=2) Quadratic Violations	r=-4 (σ=5) Translog Violations	r=-4 (σ=5) Quadratic Violations
Group1	0	10	2	12	890	2
Group2	9	22	127	52	426	40
Group3	16	8	35	10	1180	2
Group4	3	19	12	14	260	8
Group5	32	49	100	29	528	24
Group6	787	117	228	10	1358	23
Total	847	225	504	127	4642	99
Curvature Violations	No	No	Yes	No	Yes	No

Notes:
- Numbers reported are average values obtained from 1000 Monte Carlo replications. Data for quantities demanded are generated by applying Roy's identity to a CES indirect utility function. Stochastic price data are constructed as actual prices plus an error term generated from a multivariate normal distribution with mean zero and covariance matrix equal to a multiple of the actual price covariance matrix ensuring that prices remain non-negative.
- Composite food categories are as follows: 1 = meats, 2 = eggs and dairy, 3 = cereals and bread, 4 = fruits and vegetables, 5 = juice, 6 = fats and oil.