# Linear or Nonlinear Cointegration in the Purchasing Power Parity Relationship?*

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## Abstract

We test long-run PPP within a general model of cointegration of linear and nonlinear form. Nonlinear cointegration is tested with rank tests of Breitung (2001). We determine first the order of integration of each variable, using monthly data from the post-Bretton Woods era for G-10 countries. In many cases prices are I(2), whereas all exchange rates are I(1). However, there are several countries that have a price level that linearly cointegrates with the US price level so that this combination is I(1). Overall, we find some, though limited, evidence for nonlinear and also linear cointegration in the PPP model.

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**Keywords:** PPP; order of integration; nonlinear cointegration.

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# 1. Introduction

Empirical support for the theory of purchasing power parity (PPP) has been rather mixed. PPP has been tested extensively. Basically, two alternative approaches have been followed. One approach tests for a unit root in the real exchange rates, which should not have a unit root but rather be a covariance stationary process if long-run PPP holds.[1, 2] The other approach tests instead for cointegration among prices and the nominal exchange rate, which should form a stationary linear (or nonlinear) combination if long-run PPP holds. The advantage of this approach over the previous one is that it allows for a more general form of PPP where the adjustments of domestic and foreign prices need not be symmetric and proportional to the exchange rate, which is implicitly assumed in studies based on the real exchange rate.[3] Michael, Nobay and Peel (1997) allowed for nonlinear error-correction in the residuals from linear cointegration and argued that conventional linear cointegration tests ignore nonlinearities and may therefore be biased against long-run PPP. In summary, the empirical evidence with either one of the two approaches has, however, not been conclusive to date as to whether PPP holds or not.[4]

In this paper, we follow the approach based on cointegration methods. However, instead of assuming a linear cointegrating relationship as in the previous literature, we test for a general *nonlinear* form of the cointegrating relation. This is different from testing for nonlinear error-correction, or equivalently, nonlinear equilibrium-correction, towards a *linear* long-run cointegrating relation. Our analysis complements studies that have looked at nonlinear adjustment to a linearly cointegrated long-run PPP relation. We argue that a nonlinear cointegrating relationship is a plausible model for PPP and that it is therefore worthwhile to explore whether it is consistent with the data.

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[1]Most authors considered here a mean-reverting process, however, if a Balassa-Samuelson effect is present, the real exchange rate may be stationary around a deterministic time trend instead. See, among many others, Lopez, Murray and Papell (2005) for a recent study.

[2]In addition, numerous researchers estimated half-lives of deviations from PPP based on this approach. See, among others, Elliott and Pesavento (2006). A further direction of research has been to test for nonlinearites in real exchange rate movements. See, e.g., Kilian and Taylor (2003).

[3]See, for example, Cheung and Lai (1993a) who tested for linear cointegration.

[4]Other researchers attempted to resolve the issue in models of fractional cointegration (Cheung and Lai, 1993b) or in a panel cointegration framework (e.g., Pedroni, 2001) but the puzzle still remains, as Taylor and Taylor (2004) documented in a thorough survey. See also Rogoff's (1996) survey, and Lothian and Taylor (1996) for results with long spans of data.

We start with the relationship between nominal exchange rates and domestic and foreign prices and test step-by-step the necessary assumptions for long-run PPP. We do not impose a priori symmetry and proportionality in order to arrive at a real exchange rate specification. We consider in turn each G-10 country over the post-Bretton Woods floating exchange rate period. The variables involved in the PPP cointegrating relationship must have certain orders of integration in order to co-integrate. We therefore test for the order of integration first. Then, we test for nonlinear cointegration of general form by applying a test suggested by Breitung (2001). Our nonlinear framework is consistent with the theoretical model of Sercu, Uppal and Van Hulle (1995) for nonlinearities in the PPP relation due to transaction costs.

The extent to which PPP holds in the long-run is a crucial question in the context of New Open Economy Macroeconomics.[5] In this literature, one class of models incorporates sticky prices or menu costs whereas another class of models is based on international product differentiation. Complete long-run exchange rate pass-through to import prices generally holds in models with sticky prices or menu costs but does usually not hold in models with product differentiation. Complete long-run exchange rate pass-through is a necessary condition for PPP to hold.[6]

The rest of the paper proceeds as follows. Section 2 briefly discusses the methodology of the various tests for the nonlinear model of cointegration. Section 3 presents the empirical model and motivation for nonlinear cointegration. It also provides the data description and the analysis of test results for one and two unit roots and for linear and nonlinear cointegration, and, where possible, for tests of the symmetry and proportionality restrictions. Finally, Section 4 concludes.

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[5]See, among others, Betts and Devereux (2001).

[6]See Campa and Goldberg (2002) on empirical results for long-run pass-through and also Donnenfeld and Haug (2003) for a related empirical study.

# 2. Methodology

## 2.1 Rank Tests for Cointegration

In this section we will briefly discuss the method of rank tests for nonlinear cointegration proposed by Breitung (2001).[7, 8] Consider two real-valued time series $\{x_t\}_1^T$ and $\{y_t\}_1^T$ that are nonlinearly related as $y_t = f(x_t) + u_t$, where $y_t \sim I(1)$ and $f(x_t) \sim I(1)$, i.e., each series is integrated of order one. Under the null hypothesis, $u_t$ is $I(1)$ so that $y_t$ and $x_t$ are not cointegrated. Under the alternative hypothesis, $u_t$ is $I(0)$ so that $y_t$ and $x_t$ are cointegrated. The standard assumption has been that $f(x_t)$ is a linear function. However, economic theory often gives raise to nonlinear relationships so that $f(x_t)$ is assumed here to be a nonlinear function. Breitung showed that residual-based linear cointegration tests are inconsistent for some class of nonlinear functions.[9] To overcome this problem Breitung proposed tests based on the rank transformation of the time series.

Consider a slightly more general form with $u_t = g(y_t) - f(x_t)$, where $f(x_t) \sim I(1)$, $g(y_t) \sim I(1)$, and $u_t \sim I(0)$. Breitung defined a ranked series as $R_T(x_t) = $ Rank [of $x_t$ among $x_1, \ldots, x_T$], and $R_T(y_t)$ accordingly. The rank statistics are constructed by replacing $f(x_t)$ and $g(y_t)$ with the ranked series, $R_T[f(x_t)] = R_T(x_t)$ and $R_T[g(y_t)] = R_T(y_t)$. The sequence of ranks is invariant to a monotonic transformation of the data.

In general it is not known whether the functions $g(y_t)$ and $f(x_t)$ are monotonically increasing or decreasing. For this situation, Breitung proposed a two-sided test:

$$\Xi_T^* = T^{-3} \sum_{t=1}^{T} (\tilde{u}_t^R)^2 / \{\tilde{\sigma}_{\Delta u}^2\}, \tag{1}$$

with $\tilde{u}_t^R$ the least squares residuals from a regression of $R_T(y_t)$ on $R_T(x_t)$. $\tilde{\sigma}_{\Delta u}^2$ is the variance of $\Delta \tilde{u}_t^R$. Critical values for this rank test are given in Table 1 in Breitung (p. 334). The null hypothesis is rejected when the test statistic is below the critical value. The $\Xi_T^*$ test can be extended to models with three or more variables.

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[7]To be precise, Breitung's test is a test of linear cointegration under the null hypothesis against the alternative hypothesis of nonlinear cointegration.

[8]Recently, Choi and Saikkonen (2005) proposed an alternative to Breitung's testing procedures for the existence of a nonlinear cointegrating relation, a test based on sub-sampling methods for regression residuals from a specific nonlinear model. This test therefore requires the exact specification of the nonlinear model before testing can proceed.

[9]Also, see Granger and Hallman (1991).

The cointegration rank test is designed to reject the null hypothesis of no cointegration when the residuals $\tilde{u}_t^R$ are $I(0)$. Cointegration, if it exists, may be of linear form or of nonlinear form. The Monte Carlo experiments in Breitung demonstrated that the rank test has good power properties not only in the nonlinear case but also in the linear case. To decide whether a cointegrating relation is linear or nonlinear, Breitung proposed a score statistic based on the rank transformation of the time series. This test is applied if the cointegration rank test indicates cointegration.

## 2.2 Score Statistic for a Rank Test of Neglected Nonlinear Cointegration

Consider the following nonlinear relationship between two time series: $y_t = \delta_0 + \delta_1 x_t + f^*(x_t) + u_t$, where $\delta_0 + \delta_1 x_t$ is the linear part. Under the null hypothesis, $f^*(x_t) = 0$ and the $u_t$ are $I(0)$ so that there is linear cointegration. Under the alternative hypothesis, $f^*(x_t) \neq 0$ and the $u_t$ are $I(0)$ so that there is nonlinear cointegration. The score test statistic is given by $TR^2$ from a least squares regression of $\widehat{u}_t$ on $c_1 + c_2 x_t + c_3 R_T(x_t) + e_t$. The $\widehat{u}_t$ are the residuals under the null hypothesis, possibly corrected for serial correlation and endogeneity using for example the dynamic ordinary least squares method (DOLS) of Stock and Watson (1993).[10] Under the null hypothesis, the test statistic is distributed as $\chi^2$ with one degree of freedom. The extension of this test to more than two variables is straightforward.[11]

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[10] We apply the Schwarz Bayesian information criterion to select appropriate leads and lags for DOLS.

[11] Choi and Saikkonen (2004) suggested an alternative test for testing an assumed cointegrating relation for linearity against the specific alternative of a nonlinear cointegrated smooth-transition process among the variables, i.e., the variables cointegrate in a specific nonlinear form under the alternative hypothesis. This test is hence less general than Breitung's nonlinearity test. Also, Hong and Phillips (2005) proposed a further related test for linearity in cointegrated systems based on a modified RESET test using nonlinear transformations of Park and Phillips (1999, 2001). The test is based on specific finite approximations of the nonlinear relation under the alternative hypothesis, as is Choi and Saikkonen's (2004). Hong and Phillips' test is a linearity test of cointegration not only against a specific nonlinear cointegrating relation but also against the alternative of no cointegration.

# 3.  Empirical Analysis

## 3.1 The Empirical Model of PPP

In our paper we consider two linear and two nonlinear versions of the PPP relationship. The two linear versions are given by:

$$e_t = \alpha + \beta(p_t - p_t^*) + u_t \qquad \text{linear Model A} \tag{2}$$

$$e_t = \alpha + \beta_1 p_t - \beta_2 p_t^* + u_t \qquad \text{linear Model B.} \tag{3}$$

where $e_t$ is the natural logarithm of the nominal exchange rate expressed in terms of the domestic price of foreign exchange. $p_t$ and $p_t^*$ are the natural logarithm of the domestic and foreign price, respectively; $\alpha$ is a constant reflecting differences in units of measurement; $\beta$, $\beta_1$ and $\beta_2$ are positive coefficients; and $u_t$ is a covariance-stationary mean-zero error term representing the deviations from PPP.

Linear cointegration in Model A or B ensures that the variables in the model move towards a long-run PPP equilibrium. If $e_t$ and $p_t - p_t^*$ are each I(1) in Model A, and a linear combination of these variables exists that makes $u_t$ covariance-stationary, then cointegration exists and PPP holds. For Model B, we need instead $e_t$, $p_t$ and $p_t^*$ to be each I(1) and a linear combination of these variables to be I(0). It is important to note that we test in our paper for relative PPP and not absolute PPP.[12]

The restricted version of PPP in Model A imposes the symmetry restriction that the nominal exchange rate responds equally in absolute value to changes in the domestic price level and in the foreign price level. We follow here Cheung and Lai (1993a) and explicitly test for the restriction that Model A imposes relative to Model B. Cheung and Lai (1993a) argued that measurement errors make Model B a more appealing specification than Model A. Taylor (1988) and Rogoff (1996) provided further arguments why $\beta_1 = \beta_2$ may not hold.

Michael, Nobay and Peel (1997) postulated a linear cointegrating relation as in Model A and assumed a nonlinear adjustment process for the error-correction term $u_t$.[13] This means that there is a nonlinear short-run adjustment process towards the long-run equilibrium. The long-run equilibrium is represented by a linear cointegrating relationship and only the short-run correction process is nonlinear.

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[12]See, for example, Rogoff (1996).

[13]However, imposing the restrictions implied by Model A may be overly restrictive and bias results against linear cointegration in favor of nonlinear cointegration.

If we plug the real exchange rate, $q_t \equiv e_t - (p_t - p_t^*)$, into equation (2) we get:

$$q_t = \alpha + (\beta - 1)(p_t - p^*) + u_t.$$

Only with the additional assumption of long-run proportionality between exchange rates and prices, given by $\beta = 1$, do we obtain the real exchange rate as studied in the PPP literature:

$$q_t = \alpha + u_t \quad \text{restricted linear Model A.}$$

If the symmetry and proportionality restrictions are supported by the data and $e_t$ and $p_t - p_t^*$ are each I(1) and linearly cointegrated, then $q_t$ will follow a covariance-stationary and mean-reverting process and (restricted) long-run PPP will hold. Alternatively, the adjustment of the real exchange rate to long-run (restricted) PPP may follow a nonlinear path instead in this specification. Again, we will test the symmetry and proportionality restrictions, whenever possible, rather than impose them.[14]

We consider in our paper nonlinear cointegration in addition to linear cointegration. It is a nonlinear combination of the variables that renders $u_t$ covariance-stationary, if nonlinear cointegration exists. An example of a nonlinear adjustment process to long-run PPP is the quadratic form (Model A is taken for convenience):

$$e_t = \alpha + \beta(p_t - p_t^*)^2 + u_t.$$

If there is nonlinear cointegration of this form, then $u_t$ is an I(0) error-correction term that describes the adjustment process towards long-run PPP. The reaction (in percent) of the exchange rate to changes in the price ratio is given by:

$$\frac{\partial e_t}{\partial (p_t - p_t^*)} = 2\beta(p_t - p_t^*).$$

The logarithm of the nominal exchange rate adjusts faster, the larger the deviation of relative prices from long-run PPP.[15] Transformations similar to those for the linear case above show that the restriction $\beta = 1$ does here not lead to the real exchange rate model used in the literature:

$$q_t = \alpha + [\{\beta(p_t - p^*) - 1\}(p_t - p^*)] + u_t.$$

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[14]See Cheung and Lai (1993a, pp. 189–190) for further discussion.

[15]Cheung, Lai, and Bergman (2004), and Engel and Morley (2001) argued, in different types of models from ours, that nominal exchange rates, and not prices, are the "sticky" variable in the adjustment process to PPP, opposite to what had been commonly assumed previously. They also showed that the adjustment speed of exchange rates and of prices differs for adjustment towards their respective (unobservable) equilibrium values.

Sercu, Uppal and Van Hulle (1995) showed how shipping costs can lead to a band around the nominal exchange rate where no adjustment takes place when relative prices fluctuate across countries within a given range. They used a two country model with one traded good. In a multiple goods world where goods have different shipping costs and also non-traded goods are present, a scenario with adjustment speeds depending on the extent of the price differential seems more appropriate than threshold adjustment.[16] The larger the wedge between domestic and foreign prices in a given period, the larger is the number of goods with profitable arbitrage. Therefore, the more apart relative prices of two countries, the more arbitrage will take place and the higher is the speed of adjustment of the nominal exchange rate.

The general forms of the nonlinear versions of the above linear models are given by:

$$e_t = \alpha + f(p_t - p_t^*) + u_t \quad \text{nonlinear Model A} \tag{4}$$

$$e_t = \alpha + f(p_t, p_t^*) + u_t \qquad \text{nonlinear Model B.} \tag{5}$$

With respect to cointegration, we carry out Breitung's tests to determine whether $u_t$ is stationary when $f(\cdot)$ is of nonlinear form.

## 3.2 The Data and Sample Periods

Our data set is monthly and was extracted from the CD-ROM version of the IMF's *International Financial Statistics* (IFS). We use the end of period nominal exchange rate (IFS line ae), and the consumer price index (IFS line 64) for G-10 countries.[17] The sample period spans from 1973:5 to 2007:03 for Canada, Japan, Sweden, Switzerland, and the UK. The remaining five countries (Belgium, France, Germany, Italy, and the Netherlands) have joined the euro and hence the data are available from 1973:5 to 1998:12.[18]

We treated initially the United States as a numeraire country for both prices and exchange rates. Our empirical analysis is based on two sample windows. The first sample, which we refer to as "full sample", starts from 1973:5 and ends in 1998:12 for euro countries, and in 2007:03 for the other five countries. We also test the PPP

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[16]See also the discussion in Taylor and Taylor (2004, pp. 146–149).

[17]G-10 actually consists of 11 countries.

[18]The start date allows for an adjustment period following the formal end of the Bretton Woods system of fixed exchange rates with the Smithsonian Agreement in February 1973.

theory for the post-1982 sample period. Sims and Zha (2002), and Clarida, Gali and Gertler (2000) analyzed the post-1982 period separately and found a significant difference in US monetary policy. This sample spans from 1982:11 to 1998:12 for the euro countries included and from 1982:11 to 2007:03 for the other five countries. We refer to this as "sub sample". In addition, we consider German Mark based exchange rates and PPP relations for the euro countries among the group of G-10 in order to see whether our results are sensitive to the choice of numeraire country.

## 3.3 Full Sample Analysis for US Dollar Based Exchange Rates

First, we test the order of integration of the variables that enter Models A and B. Following the suggestion of Dickey and Pantula (1987), we start with testing for two unit roots or I(2) because there is some empirical evidence in the literature suggesting that the natural logarithm of prices may be I(2) and inflation henceforth I(1). The null hypothesis is two unit roots (or one unit root in the first-differenced variable) against the alternative hypothesis of I(1). If the null hypothesis of two unit roots is rejected, we test the null hypothesis of I(1) against the alternative of covariance-stationarity next. As is well known, unit root tests may have substantial size distortions when there are large negative moving average components in the data generating process.[19] We use state-of-the-art unit root tests that have been shown to have good size and power properties in such situations.[20] We base our inference on the 5% level of significance. However, for p-values that fall in the range of .05 and .10, we carried out a Box-Jenkins ARIMA analysis in addition.

We apply the DF-GLS test of Elliott, Rothenberg and Stock (1996) for our unit root tests. When testing for I(1), we allow for a constant but no deterministic time trends in the test regression, except for prices. We consider for prices ($p_t$, $p_t^*$, and $p_t^{'}$) two cases, one with a constant and the other with a constant and a deterministic time trend in the test regression.[21] The first case implies that inflation, which is equal to $\Delta p_t$, has no constant. The second case implies that inflation has a constant. The DF-GLS test procedure applies the Dickey-Fuller $\tau$-test to locally demeaned or demeaned and detrended series. It has generally higher power than the standard

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[19]See Schwert (1989).

[20]See Ng and Perron (2001).

[21]$p_t^{'}$ denotes the German price level, in logarithms.

Augmented Dickey-Fuller (ADF) unit root test. Ng and Perron (2001) studied the size and power properties of the DF-GLS test in typical finite samples and recommended using a modified Akaike criterion (MAIC) in order to select the lag length in the test regressions. We follow this suggestion and use the DF-GLS test with MAIC as implemented in the software package EViews 5.1. When testing for two unit roots without a constant term in the test regression, it is unnecessary to locally demean the series. We therefore apply the standard ADF test in this case, again in connection with MAIC. The DF-GLS test with a constant only has the same limiting distribution as the ADF test without a constant. We use the program of MacKinnon (1996) to calculate p-values. The DF-GLS test with a constant and deterministic time trend has a nonstandard distribution for which the critical values are provided in Elliott et al. (1996).

Table 1 reports p-values for the tests for two roots and for one unit root. The results for $\Delta(p_t - p_t^*)$ reveal that $p_t - p_t^*$ has two unit roots for 6 countries because the null hypothesis of I(2) cannot be rejected at the 5% significance level. The null hypothesis of I(2) is rejected for the remaining 4 countries (as indicated by bold figures in the tables). The p-values are between 0.05 and 0.10 for 5 of the 6 countries. We carried out an ARIMA analysis for these 5 countries and found very slowly and linearly decaying autocorrelations for the series in first-differences. Also, the partial autocorrelations showed a spike with a value near one.[22] All this evidence points to an I(2) process.

The null hypothesis of two unit roots for $p_t$ with a constant in the test regression, labelled $\Delta p_t$ in Table 1, is not rejected for 6 countries and is also not rejected for $p_t^*$, the US price level. This evidence is consistent with many other studies that found inflation to be well approximated by an I(1) process for most countries. If inflation, defined as the first difference of the log of prices, is I(1), then prices must be I(2). A recent example of evidence for I(1) of inflation is Rapach (2003). He (p. 30) argued that the DF and DF-GLS tests "convincingly indicate" that inflation is I(1) for each of the 14 OECD countries that he studied.[23,24] I(2) is rejected for the remaining 4

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[22]Results are not reported to conserve space but are available form the authors on request.

[23]See also Banerjee, Cockerell and Russel (2001), among others, who reported evidence for I(2) of prices. Also, Bacchiocchi and Fanelli (2005) include the inflation rate in the PPP equation as an I(1) variable which allows them to account for I(1) and I(2) processes of the other variables through polynomial cointegration.

[24]If $p_t$ and $p_t^*$ are individually I(2), then the difference of the two series is I(2), unless they

of the 10 countries in our study. When we exclude the constant term from the test regression for two unit roots, I(2) is not rejected for prices for 3 countries and is a borderline case for the US (p=0.047), though the ARIMA analysis supports I(2). Two unit roots are clearly rejected for all countries in the case of the nominal exchange rate and results are not reported to conserve space.

We proceed to testing of the I(1) hypothesis for the countries for which we were able to reject two unit roots.[25] We cannot reject the hypothesis of a unit root for all exchange rates, for $p_t - p_t^*$ for all the 4 countries with rejections of I(2) (we did not reject I(2) for Belgium), and for $p_t$ for all countries considered.[26]

The variables that enter (linear or nonlinear) Model A have for the majority of countries different orders of integration. For (linear or nonlinear) Model B the base country is the US and its price level, $p_t^*$, follows an I(2) process which is different from the process for the exchange rate. This result implies that some price levels, those that are I(2), are driven by different permanent shocks than the exchange rate, which is for all countries I(1).

A long-run equilibrium in the form of cointegration in Models A and B can only exist if the variables have the same order of integration. This implies that we can only test for PPP in Model A for Canada, Japan, Sweden and the UK. Model B is ruled out because the US has a price level that is I(2) whereas all exchange rates are I(1), unless $p_t$ and $p_t^*$ are each individually I(2) and form a cointergating relation such that $\beta_1 p_t - \beta_2 p_t^*$ is I(1).[27] Equivalently, $p_t - \frac{\beta_2}{\beta_1} p_t^*$ is then I(1). PPP implies that this term should be cointergated with the exchange rate, which is possible because both are I(1). In this case, we should find two cointegrating vectors for Model B with the Johansen method.

We first consider linear and then nonlinear cointegration. The trace test of Johansen (1995) for linear cointegration, reported in Table 1, leads to a rejection of the null hypothesis of no cointegration for Japan and the UK for Model A.[28] On

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cointegrate to I(1) with cointegrating parameters of 1 and -1. If one price is I(2) and the other is I(1), then the difference is I(2), unless they cointegrate to I(1) with parameters of 1 and -1.

[25]We report test results for one unit root for all countries for completeness only. The procedure of Dickey and Pantula (1987) is to stop further testing once the null hypothesis of I(2) is not rejected.

[26]We do not report the results for $p_t$ for the tests with a constant and time trend, however, a unit root cannot be rejected for all countries, based on the DF-GLS test.

[27]We thank Peter Ireland for pointing out this possibility to us.

[28]See, for example, Haug (1996) on Monte Carlo evidence for the performance of various cointegration tests in small samples.

the other hand, this hypothesis is not rejected for Canada and Sweden.[29] For Model A, this leaves us with only two countries with linear cointegration, Japan and the UK. We find evidence for one cointegrating vector for each country. However, the cointegrating vector for Japan has the wrong sign so that we are left with only the UK.[30]

Prices are likely I(2) for Canada, France, Germany, Italy, Sweden, Switzerland, and the US, as can be seen in Table 1 from the column labelled $\Delta p_t$ and from the Notes for the US. We therefore test for cointegration in Model B. We report in Table 1 results for those countries that produce two cointegrating vectors as required by PPP: France, Italy and the UK. The estimated parameters in the cointegrating vectors are consistent with theory. That means that the domestic and US price form a cointegrating relationship that is I(1) and in turn cointegrates with the exchange rate so that Model B is supported. Overall, this provides some limited support for PPP as formulated in Model B.

Would possibly nonlinear cointegration lead to more results in favor of cointegration? We first apply Breitung's (2001) test for linear or nonlinear cointegration, the $\Xi_T^*$ test, for which the null hypothesis of no cointegration is rejected when the test statistic takes on a value below the critical value at a given significance level. As Table 2 shows, we reject the null hypothesis at the 1% level for all 4 countries that have the same order of integration for individual variables in nonlinear Model A. This test does not tell whether cointegration is of linear or of nonlinear form. We apply Breitung's nonlinear score test for this purpose. Results in Table 2 clearly indicate that all cointegration is of linear form for Model A. The same is true for Model B for the chosen countries, except for the UK for which nonlinear cointegration is supported, based on 5% levels of significance.[31] We therefore rely for the linear cases on the above results with the Johansen test that has a narrower alternative hypothesis and therefore more power. In summary, we have uncovered some very limited evidence for linear cointegration and therefore for long-run PPP over the full sample period.

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[29]The possibly more powerful maximum eigenvalue test of Johansen (1995) leads to the same results.

[30]For our definition of the exchange rate, $\beta$ should be positive in equation (2). Also, we tested the hypothesis that $\beta = 1$ for the UK but strongly rejected this hypothesis with a p-value of zero for the likelihood ratio test.

[31]We tested for the UK the restrictions on linear Model B that imply linear Model A but these were rejected. This indicates a misspecified model for the UK because we found support for linear Model A but then found nonlinearities in Model B.

## 3.4 Sub Sample Analysis for US Dollar Based Exchange Rates

We consider now the period from 1982:11 onwards, as motivated in Section 3.2. We follow numerous other empirical studies that documented a change in US monetary policy at that time. We first test for the order of integration and then for linear and nonlinear cointegration.

We report unit root test results for the shortened sample in Table 3. The ADF test with MAIC suggests two unit roots in $p_t - p_t^*$ for 4 countries, compared to 6 countries over the full sample in the previous section.[32] The tests indicate two unit roots for the price level, $p_t$, for 9 countries when a constant is present and for 6 countries without the constant term. In addition, the US price level, $p_t^*$, has clearly two unit roots with and without the constant term. Also, we again strongly reject two unit roots for the nominal exchange rate for all countries (results are not reported). The DF-GLS tests cannot reject one unit root for all variables, at the 5% level of significance, except for the exchange rate of the UK. The exchange rate of the UK therefore seems to be I(0).[33]

The variables in Model A have the same order of integration for Belgium, Canada, France, Italy, and Sweden. Hence, it is possible to test for linear and nonlinear cointegration for these countries within Model A. Model B is again ruled out because the price level of the US is I(2), as before over the full sample, unless the domestic price level is I(2) and cointegrates with the US price level that is I(2).

The Johansen tests in Table 3 for Model A clearly reject the null hypothesis of no linear cointegration for Belgium, France and Italy but not for Canada and Sweden. However, all estimated values of $\beta$ have the incorrect sign for PPP. Hence, we do not find any linear Model A that supports long-run PPP.

The evidence for Model B looks somewhat more promising. Prices are found to be I(2) for all countries, except for Belgium. We test for cointegration in Model B for these countries. We report in Table 3 results for the countries that produce two cointegrating vectors as required by PPP: France, Italy and Sweden. The estimated parameters in the cointegrating vectors are again consistent with theory. This provides some limited support for PPP in the form of Model B.

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[32]See the column labelled $\Delta(p_t - p_t^*)$ in Table 3.

[33]We cannot reject one unit root for $p_t$ and $p_t^*$ either when the test regression includes a constant and deterministic time trend (not reported).

Table 4 presents test results for Breitung's tests of nonlinear cointegration over the sub sample. For Model A, we reject the null hypothesis of no cointegration in favor of cointegration of either linear or nonlinear form for all 5 countries considered. The nonlinear score test indicates linear cointegration, except for Italy, taking a 5% level of significance. For Model B, we find linear cointegration for France and Italy, and nonlinear cointegration for Sweden.

Our analysis over the sub sample leaves us therefore with one case that support long-run PPP in Model A: a nonlinear cointegrating relation for Italy. There are three cases of support for cointegration in Model B: linear cointegration for France and Italy and nonlinear cointegration for Sweden. It is interesting to see that the result for France and Italy is robust across the full and sub sample. Also, Model A fits in nonlinear form for Italy and in linear form for Model B. This is not a contradiction and rather lends additional support to PPP in the case of Italy.

## 3.5 Analysis for German Mark Based Exchange Rates for Euro Countries

If PPP is likely to hold for any set of countries, it should be for the countries that adopted the euro. We use the German Mark as the base currency for the exchange rate and apply the various tests to Belgium, France, Italy, and the Netherlands, which are the euro countries among the G-10. We use the monthly observations from 1973:05 to 1998:12.

Results are reported in Tables 5 and 6. Two unit roots in $p_t - p_t^{'}$ are rejected for Belgium and the Netherlands but not for France and Italy. Further, two unit roots in $p_t$ can be rejected for all euro countries in Table 5 only for the case without a constant.[34] When a constant term is present, the null hypothesis of I(2) is not rejected for France, Italy, and Germany. The test results for two unit roots in $p_t^{'}$ are borderline cases. ARIMA analysis does not provide clear guidance either. For France and Italy, we require two cointegrating vectors for Model B. On the other hand, for Belgium and the Netherland, we require at least one cointegrating vector in Model B. Next, the tests for unit roots suggest that $p_t - p_t^{'}$ and the price level are I(1) for Belgium and the Netherlands, be it with a constant only or with a constant and time

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[34]Results from Table 1 are repeated in Table 5 for convenience.

trend (not reported) in the regression for $p_t$. The German Mark based exchange rates, $e_t^{'}$, are I(1) as well for all countries.

We test for linear cointegration within Model A and B for Belgium and the Netherlands. The Johansen tests in Table 5 clearly reject the null hypothesis of no cointegration in all cases. However, we find two cointegrating vectors for the Netherlands for Models A and for Model B. This means in Model A that all variables should be I(0), which contradicts our unit root test results. We therefore dismiss Model A for the Netherlands. On the other hand, two cointegrating vectors in Model B are not a problem. However, none of the estimated cointegrating vectors for all Models A and B in Table 5 has the correct signs as required by PPP. We also tested for linear cointegration in Model B for France and Italy and did not find support for two cointegrating vectors (not reported) so that Model B is not supported for these two countries either.

Next, we apply the tests for nonlinear cointegration. We find evidence for cointegration of linear or nonlinear form for Belgium and the Netherlands for Models A and B. The score test indicates only for Belgium that cointegration is of nonlinear form in Model B. The analysis of the euro countries therefore leaves us with only one case that supports PPP: the nonlinear Model B of Belgium.

## 4. Conclusion

In this paper, we considered the possibility of a nonlinear cointegrating relationship for the purchasing power parity (PPP) model. We applied for this purpose recently developed direct tests for nonlinear cointegration based on ranked time series. We allowed for non-symmetric price adjustment and for non-proportional movements of prices and exchange rates in the long-run PPP relationship. We have argued that a PPP-model with nonlinear cointegration is a plausible alternative to the linear cointegration model used in the literature.

We examined the nonlinear PPP relation over the post-Bretton Woods floating exchange rate period for the G-10 countries. We considered US dollar based exchange rates for a sample of monthly observations starting in 1973:05 and for another sample

starting in 1982:11 instead. We also considered German Mark based exchange rates for those G-10 countries that adopted the euro.

We found evidence for integration of order two for the price levels for over two-thirds, and for the domestic to foreign price ratios for almost half of all the different specifications considered in our analysis. On the other hand, all nominal exchange rates are integrated of order one.

In our analysis, we found only limited evidence in favor of linear and nonlinear cointegration as required by long-run PPP, despite using a very general specification. Linear PPP is supported over the full sample period for France and Italy.[35] Overall, the sub period from 1982:11 onwards provides more support for PPP than the full sample starting in 1973:05. The sub period shows support for PPP of nonlinear form for Belgium, Italy and Sweden, and of linear form for France and also for Italy if a more general specification is used for the latter. Our results for the euro countries (Belgium, France and Italy) are unfortunately sensitive to the base currency used for the exchange rate.

In summary, our results provide mixed support for long-run PPP: 4 out of 10 countries show the required cointegrating relationships, of which at least half are nonlinear. In future research, it would be worthwhile to explore further to what extend PPP fails. Even though it is formally rejected in many cases, a general specification of PPP as discussed in this paper may still provide a very close approximation to the data.

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[35]See also Hong and Phillips (2005) for a finding of no support for linear cointegration with a different test applied to Canada and Japan.

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**Table 1.  US Dollar Based Analysis:  P-Values of Unit Root and Johansen Cointegration Tests for the Full Sample**

| Countries | Unit Root Tests† | | | | | | Johansen (Trace) Cointegration Test‡ | |
|---|---|---|---|---|---|---|---|---|
| | $\Delta (p_t - p^*_t)$ no constant | $\Delta p_t$ | $\Delta p_t$ no constant | $p_t - p^*_t$ | $p_t$ | $e_t$ | Model A | Model B |
| Belgium | 0.082 | **0.013** | **0.014** | **0.036** | 0.778 | 0.132 | - | - |
| Canada | **0.000** | 0.452 | 0.067 | 0.326 | 0.833 | 0.488 | 0.951 | - |
| France | 0.055 | 0.663 | **0.029** | 0.424 | 0.611 | 0.340 | - | **0.000**◊ |
| Germany | 0.062 | 0.097 | **0.047** | 0.792 | 0.866 | 0.540 | - | - |
| Italy | 0.093 | 0.143 | **0.025** | 0.738 | 0.761 | 0.807 | - | **0.000**◊ |
| Japan | **0.001** | **0.000** | **0.000** | 0.958 | 0.920 | 0.652 | **0.000** | - |
| Netherlands | 0.102 | **0.015** | **0.029** | 0.703 | 0.842 | 0.447 | - | - |
| Sweden | **0.002** | 0.459 | 0.145 | 0.536 | 0.731 | 0.509 | 0.382 | - |
| Switzerland | 0.094 | 0.554 | **0.005** | 0.850 | 0.909 | 0.909 | - | - |
| UK | **0.019** | **0.044** | 0.052 | 0.806 | 0.878 | 0.427 | **0.000** | **0.000**◊ |

Notes:  The sample period is from 1973:05 to 2007:03 or 1998:12 for euro countries.
† We apply the DF-GLS unit root test of Elliot, Rothenberg and Stock (1996).  The lag length is chosen using the modified Akaike's criterion (MAIC) suggested by Ng and Perron (2001).  We test for a unit root in the levels data with the DF-GLS test that includes a constant in the test regression but no deterministic time trends.  We use this test also for the variables in first differences with a constant included. When there is no constant term in the unit root test regression for the variables in first differences, the ADF test with MAIC is applied because the DF-GLS test is identical to the ADF test in this case.  The maximum number of lags has been fixed at 17 (15) for G-5 (euro) countries. A value of .000 indicates a p-value of less than .0005.  P-values are calculated with the program of MacKinnon (1996).  The p-value for two unit roots in the US price level, p*$_t$, is 0.450 (0.047 with no constant) for the sample of the G-5 countries, and 0.445 (0.073 with no constant) for the sample period of the euro countries.  For the test of one unit root, it is 0.881 and 0.777, for the sample period of the G-5 and the euro countries respectively.  The p-values for the logarithm of the nominal exchange rate, $e_t$, for two unit roots reject the null hypothesis for all countries and are not reported. Bold figures indicate rejection at the 5% level of significance.
‡ For Johansen cointegration tests we allow for a constant but no time trend in the cointegrating regression and VAR. The number of lags is chosen using the Schwarz Bayesian criterion. The p-values for Johansen tests are computed using the program of MacKinnon, Haug, and Michelis (1999).
◊ Tests indicate that there are two cointegrating vectors in the system.


**Table 2.  US Dollar Based Analysis:  Rank Tests of Nonlinear Cointegration for the Full Sample**

| Countries | Model A | | Model B | |
|---|---|---|---|---|
| | $\Xi^*_T$ | Nonlinear Score Test | $\Xi^*_T$ | Nonlinear Score Test |
| Belgium | - | - | - | - |
| Canada | 0.002[***] | 1.303 | - | - |
| France | - | - | 0.003[***] | 2.767[*] |
| Germany | - | - | - | - |
| Italy | - | - | 0.003[***] | 3.015[*] |
| Japan | 0.002[***] | 0.157 | - | - |
| Netherlands | - | - | - | - |
| Sweden | 0.002[***] | 0.076 | - | - |
| Switzerland | - | - | - | - |
| UK | 0.003[***] | 0.035 | 0.002[***] | 5.011[**] |

Notes:  The sample period is from 1973:05 to 2007:03 or 1998:12 for euro countries. Significance at the 1%, 5%, and 10% level is indicated by [***], [**], [*].  Critical values for the $\Xi^*_T$ test statistic are from Breitung (2001, Table 1).  The 1% critical value is 0.0136 for Model A and .0119 for Model B.  The null hypothesis of no nonlinear cointegration is rejected for a test statistic value *smaller* than the critical value.  The nonlinear-score test follows a $\chi^2$ distribution with one degree of freedom for Model A and two degrees of freedom for Model B.

**Table 3. US Dollar Based Analysis: P-Values of Unit Root and Johansen Cointegration Tests for the Sub Sample**

| Countries | Unit Root Tests† | | | | | | Johansen (Trace) Cointegration Test | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| | $\Delta (p_t - p^*_t)$ no constant | $\Delta p_t$ | $\Delta p_t$ no constant | $p_t - p^*_t$ | $p_t$ | $e_t$ | Model A | Model B |
| Belgium | **0.031** | **0.034** | **0.019** | 0.364 | 0.884 | 0.498 | **0.001** | - |
| Canada | **0.000** | 0.718 | 0.112 | 0.635 | 0.927 | 0.265 | 0.994 | - |
| France | **0.003** | 0.870 | **0.002** | 0.300 | 0.893 | 0.324 | **0.000** | **0.000**$^\diamond$ |
| Germany | 0.053 | 0.061 | 0.112 | 0.757 | 0.819 | 0.548 | - | - |
| Italy | **0.000** | 0.409 | **0.019** | 0.873 | 0.909 | 0.157 | **0.000** | **0.000**$^\diamond$ |
| Japan | 0.185 | 0.696 | **0.041** | 0.978 | 0.672 | 0692 | - | - |
| Netherlands | 0.164 | 0.285 | 0.202 | 0.632 | 0.876 | 0.523 | - | - |
| Sweden | **0.009** | 0.703 | 0.063 | 0.432 | 0.803 | 0.058 | 0.340 | **0.000**$^\diamond$ |
| Switzerland | 0.152 | 0.735 | 0.167 | 0.982 | 0.617 | 0.571 | - | - |

Notes: See Table 1. The sample period is from 1982:11 to 2007:03 or 1998:12 for euro countries. The maximum number of lags has been fixed at 15 (14) for G-5 (euro) countries. For the US price level, $p^*_t$, the p-value for the test for two unit roots is 0.367 (0.205 with no constant) for the sample period of the G-5 countries, and 0.357 (0.249 with no constant) for the sample period of the euro countries. The p-value for the DF-GLS test for one unit root is 0.911 and 0.730, for G-5 and euro countries respectively.


**Table 4. US Dollar Based Analysis: Rank Tests of Nonlinear Cointegration for the Sub Sample**

| Countries | Model A | | Model B | |
| --- | --- | --- | --- | --- |
| | $\Xi^*_T$ | Nonlinear Score Test | $\Xi^*_T$ | Nonlinear Score Test |
| Belgium | 0.005*** | 1.787 | - | - |
| Canada | 0.003*** | 0.131 | - | - |
| France | 0.006*** | 0.099 | 0.005*** | 0.221 |
| Germany | - | - | - | - |
| Italy | 0.005*** | 5.874** | 0.005*** | 0.524 |
| Japan | - | - | - | - |
| Netherlands | - | - | - | - |
| Sweden | 0.003*** | 0.347 | 0.003*** | 13.5533*** |
| Switzerland | - | - | - | - |
| UK | - | - | - | - |

Notes: See Table 2.

**Table 5.  German Mark Based Analysis: P-Values of Unit Root and Johansen Cointegration Tests for the Euro Countries for the Sub Sample**

| Countries | Unit Root Tests† | | | | | | Johansen (Trace) Cointegration Test‡ | |
|---|---|---|---|---|---|---|---|---|
| | $\Delta\,(p_t - p'_t)$ no constant | $\Delta\,p_t$ | $\Delta\,p_t$ no constant | $p_t - p'_t$ | $p_t$ | $e'_t$ | Model A | Model B |
| Belgium | **0.010** | **0.013** | **0.014** | 0.718 | 0.778 | 0.896 | **0.000** | **0.000** |
| France | 0.099 | 0.663 | **0.029** | 0.650 | 0.611 | 0.834 | - | - |
| Italy | 0.103 | 0.143 | **0.025** | 0.767 | 0.761 | 0.981 | - | - |
| Netherlands | **0.016** | **0.015** | 0.029 | 0.653 | 0.842 | 0.798 | **0.000**$^\diamond$ | **0.000**$^\diamond$ |

Notes: See Table 1.  The sample period is from 1982:11 to 1998:12.  The hypothesis of two unit roots is rejected for all exchange rates.  The p-value for the German price level, $p'_t$, is 0.097 (0.047 with no constant) for the test for two unit roots and 0.866 for the test for one root.


**Table 6.  German Mark Based Analysis: Rank Tests of Nonlinear Cointegration for the Euro Countries for the Sub Sample**

| Countries | Model A | | Model B | |
|---|---|---|---|---|
| | $\Xi^*_T$ | Nonlinear Score Test | $\Xi^*_T$ | Nonlinear Score Test |
| Belgium | 0.0038[***] | 0.503 | 0.0035[***] | 13.350[***] |
| France | - | - | - | - |
| Italy | - | - | - | - |
| Netherlands | 0.0036[***] | 1.891 | 0.0038[***] | 0.165 |

Notes: See Table 2.  The sample period is from 1982:11 to 1998:12.