Peter C. B. Phillips' Contributions and Influence in Financial Econometrics

Jun Yu[1]

[1]Singapore Management University

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Outline

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Overview 1: Indirect influence

1. Unit root and cointegration in financial time series.

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5. Nonlinear cointegration. References: Phillips and Park (2002). Financial Applications: Hu and Phillips (2004), Huang and Yu (2008).

Overview 2: Direct influence

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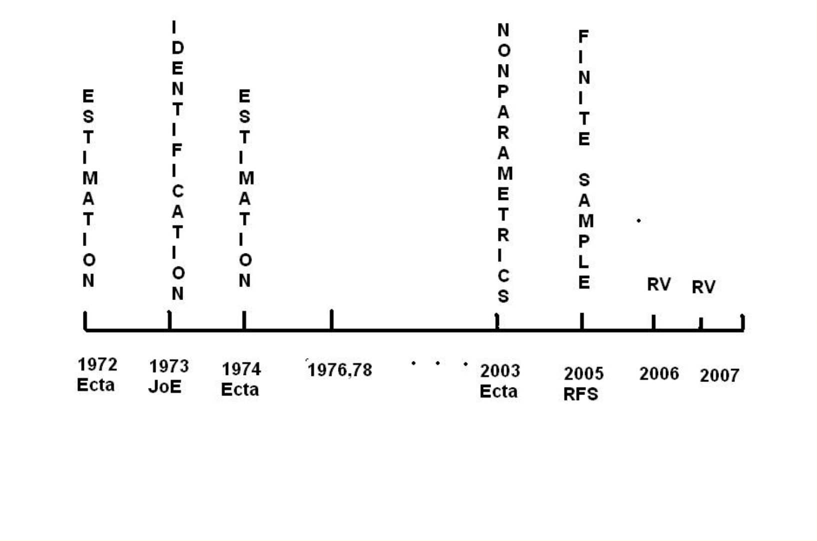
5. Others

Overview 3: Time Series Plot



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- First study of identification of continuous time econometric models.

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- The exact discrete time model is:

$$X_t = BX_{t-1} + \varepsilon_t, \tag{3}$$

where $B = \exp(Ah)$, $\varepsilon_t = \int_0^h \exp(sA)\xi(t-s)ds$.

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- Extensions: Hansen and Sargent (1983, Ecta).

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- Why exact discrete time model? Not subject to discretization bias!
- Euler discretization:
$$X_t = (1 - Ah)X_{t-1} + \epsilon_t.$$

Continue

- When $A = -1$, $h = 1/12$, e^{Ah} is 0.92 whereas $1 - Ah$ is 0.9167. If $A = 1$, $h = 1$, then e^{Ah} is 0.3679 whereas $1 - Ah$ is 0.

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- GLS procedure for the exact discrete time model:

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- Gaussian estimation (QMLE).

The Extensions: Phillips (1974, Ecta)

- Extension 1 – *model with identities*

$$DX(t) = A(\theta)X(t) + H\xi(t),$$

where A is $n \times n$ but H is $n \times r$ with $r < n$.

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- Conditions are needed to ensure the positivity of the covariance of ε_t.

- Extension 2 – *model with exogenous variables*

$$DX(t) = A(\theta h)X(t) + BZ(t) + H\xi(t),$$

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- Further extensions: mixed stock and flow variables, higher order models.

Interest Rate Models: Yu and Phillips (2001, Ects J)

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- The error is not normal.

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- To get an exact discrete model with Gaussian errors, Yu and Phillips (2001) used

$$r(t_{j+1}) = \mu(1 - e^{-\kappa h_{j+1}}) + e^{-\kappa h_{j+1}} r(t_j) + M(h_{j+1}), \qquad (6)$$

where $M(h)$ is a continuous martingale with quadratic variation

$$[M]_h = \sigma^2 \int_0^h e^{-2\kappa(h-\tau)} \sigma^2(t+\tau) d\tau, \qquad (7)$$

$$h_{j+1} = \inf\{s | [M_j]_s \geq a\} = \inf\{s | \int_0^s e^{-2\kappa(s-\tau)} \sigma^2(t_j+\tau) d\tau \geq a\}, \qquad (8)$$

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- According to Dambis, Dambus-Schwarz theorem, $M(h_{j+1}) \sim N(0, a)$.

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- Assume X_t is observed at equispaced points $(\Delta_{n,T}, 2\Delta_{n,T}, \cdots, n\Delta_{n,T})$ over the period $[0, T]$. So $n = T\Delta_{n,T}$. It is assumed $n \to \infty$, $T \to \infty$ and $\Delta_{n,T} = T/n \to 0$.

Continue

- The nonparametric estimator of $\mu(X(t))$ is

$$\frac{\sum_{i=1}^{n} K\left(\frac{X_{i\Delta_{n,T}}-x}{h_{n,t}}\right)\tilde{\mu}_{n,T}(x)}{\sum_{i=1}^{n} K\left(\frac{X_{i\Delta_{n,T}}-x}{h_{n,t}}\right)}$$

- The nonparametric estimator of $\sigma^2(X(t))$ is

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$$\tilde{\mu}_{n,T}(x) = \frac{1}{m_{n,T}(i\Delta_{n,T})\Delta_{n,T}} \sum_{j=0}^{m_{n,T}(i\Delta_{n,T})-1} \left[X_{t(i\Delta_{n,T})_j+\Delta_{n,T}} - X_{t(i\Delta_{n,T})_j}\right]$$

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- Application 2: Corradi and Disto (2007)
- Application 3: Jeffrey, Linton, Nguyen, Phillips (2004)

Method 2: Bandi and Phillips (2006, JoE)

- Estimate parametric models via nonparametric methods

$$dX(t) = \mu(X(t);\theta_1)dt + \sigma(X(t);\theta_2)dB(t), \tag{12}$$

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Method 3: Phillips and Yu, (JoE, forthcoming)

- Estimate parametric models via nonparametric and parametric methods

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$$dX(t) = \mu(X(t);\theta_1)dt + \sigma(X(t);\theta_2)dB(t),$$

- If the diffusion term is known (i.e. $\sigma(X_t;\theta_2) = \sigma(X_t)$), the exact continuous record log-likelihood is

$$\ell_{IF}(\theta_1) = \int_0^T \frac{\mu(X_t;\theta_1)}{\sigma^2(X_t)} dX_t - \frac{1}{2}\int_0^T \frac{\mu^2(X_t;\theta_1)}{\sigma^2(X_t)} dt.$$

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- On the other hand

$$[X]_T = \int_0^T (dX_t)^2 = \int_0^T \sigma^2(X_t;\theta_2)dt,$$

where $[X]_T$ is the quadratic variation of X, which can be consistently estimated by $[X_\Delta]_T$ as $\Delta \to 0$.

Method 4: Phillips and Yu (2006)

- Estimate integrated volatility using nonparametric methods

$$dp^*(t) = \sigma(t)dB(t), \tag{13}$$

where $p^*(t)$ is the equilibrium log-price and $\sigma(t)$ is a càdlàg volatility process. The quantity of interest is $IV = \int_0^1 \sigma^2(t)dt$.

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$$\sum_{i=1}^{m} [p^*_{i,m} - p^*_{i-1,m}]^2 := RV^{(m)}(p^*).$$

- Barndorff-Nielsen and Shephard (2002) showed that

$$\sqrt{m}\left[RV^{(m)}(p^*) - IV\right] |\sigma^2(t) \xrightarrow{d} MN\left(0, 2\int_0^1 \sigma^4(t)dt\right). \tag{14}$$

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$$p_{i,m} = \begin{cases} p^*_{i,m} & \text{if } \xi_i = 1 \\ p_{i-1,m} & \text{if } \xi_i = 0 \end{cases}, \tag{15}$$

where ξ_i is a Bernoulli sequence with $E(\xi_i = 1) = \pi$.

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- New model

$$p_{i,m} = \begin{cases} p^*_{i,m} & \text{if } \xi_i = 1 \\ p_{i-1,m} & \text{if } \xi_i = 0 \end{cases}, \qquad (15)$$

where ξ_i is a Bernoulli sequence with $E(\xi_i = 1) = \pi$.

- New CLT

$$\sqrt{m}\left[RV^{(m)}(p) - IV\right] \xrightarrow{d} MN\left(0, \frac{4-2\pi}{\pi}\int_0^1 \sigma^4(t)dt\right), \qquad (16)$$

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- Closed-form approximations of transition density (Aït-Sahalia, 2002, Aït-Sahalia and Yu, 2006).

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Exact and Approximate ML Estimation						
True Value $\kappa = 0.1$						
	Exact	Euler	Milst	Nowman	Infill	Herm
Mean	.2403	.2419	.2444	.2386	.2419	.2400
Std	.2777	.2867	.2867	.2771	.2867	.2762
RMSE	.3112	.3199	.3210	.3098	.3199	.3096
Mean of diff	NA	.0016	.0041	-.0017	.0016	**-.0003**
Std of diff	NA	.0500	.0453	.0162	.0500	**.0043**

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Parameter		κ	Bond Price	Option Price
True Value		**0.1**	**0.8503**	**2.3920**
Exact ML of CIR	Mean	0.1845	0.8438	1.8085
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Euler ML of CIR	Mean	0.1905	0.8433	1.7693
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- Bias from nonlinear plug-in

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- Second source of bias – nonlinearity in pricing

Jackknife, Phillips and Yu, 2005, RFS

- Let N be the number of observations in the whole sample and decompose the sample into m consecutive subsamples each with ℓ observations, so that $N = m \times \ell$. Phillips and Yu (2005a) proposed the following jackknife estimator of κ, estimator

$$\hat{\kappa}_{jack} = \frac{m}{m-1}\hat{\kappa}_N - \frac{\sum_{i=1}^{m} \hat{\kappa}_{li}}{m^2 - m}, \tag{19}$$

Continue

- To reduce the bias in pricing, jackknife prices directly

$$\hat{P}_{jack} = 2\hat{P}_T - \frac{1}{2}(\hat{P}_{1,T/2} + \hat{P}_{2,T/2}).$$

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- The jackknife almost completely removed the bias in $\hat{\theta}_T^{ML}$.
- The jackknife reduced the bias in option prices in ML down to 10-15%.

Simulation Methods, Phillips and Yu, 2007

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Outline

Other Contributions

1. Model with time-varying variances. References: Phillips and Xu (2006, 2007).

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4. Federal funds target rates. Reference: Hu and Phillips (2004).

Outline

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